FORECASTING MINIMUM TEMPERATURES ON CLEAR WINTER NIGHTS IN AN ARID REGION

A Comparison of Several Climatological Aids

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ABSTRACT

Minimum temperature formulas for clear nights in December, January, and February are developed by the Young method and tested on both original and test data. The results of these tests lead to a discussion of some weaknesses of the "method of arbitrary corrections." Jacobs' graphical adaptation of Brunt's equation is tested (a) using experimentally determined values for the local soil constants to compute the "effective" soil factor, and (b) using a soil factor determined empirically for local meteorological data. With Brunt's equation as a model, the physical justification for Young's method is discussed, and a more direct approach suggested using the evening dry bulb and wet bulb temperatures and a modern method of data analysis. There is further discussion of some implicit assumptions in the Young method of analysis and an attempt is made to see if these assumptions are satisfied by the analysis performed in combining the evening dry bulb and wet bulb temperatures and the expected morning minimum using modern methods. In the Appendix, the application of the latter method is extended to cloudy nights and performance comparisons with official forecasts are made and discussed.

1. INTRODUCTION

Meteorologists in many parts of the world have concerned themselves during the past half century with the problem of forecasting the minimum temperature using hygrometric data [13]. Most of the methods apply best on clear nights, and attempt to provide the forecaster with an estimate of the fall of temperature to be expected during the night (under average radiation conditions), using surface temperature and hygrometric readings near the time of sunset. Most of the methods do not provide a completely objective forecast, but encourage the forecaster to modify the formula value subjectively on the basis of expected wind and sky conditions during the night; hence, they may be called "climatological aids" to forecasting.

In the Salt River Valley of Arizona, minimum temperatures are of major concern to citrus growers during the winter season. Many of the conditions required for the successful functioning of most of the formulas are satisfied locally on the majority of winter nights because (1) the skies are usually clear, (2) surface winds are light, and (3) the air is relatively free from pollutants. Hence, this should be a good place to compare the performance of hygrometric relationships.

Many minimum temperature studies in the past have been concerned only with temperatures in the lower ranges, since those are the cases of primary interest in frost forecasting. The present study is made with the forecasting program of a First Order Weather Bureau Station in mind, and purposely includes temperatures in all ranges. Daily forecasts for such a station are given press, radio, and television distribution, so that minimum temperature forecasts in higher ranges, too, are of considerable operational importance.

TABLE 1.—*Data periods used in the study.*

Original Data			Test Data		
Month	Years included	No. cases	Month	Years included	No. cases
Dec	1948, '50, '52, '54, '56	98	Dec	1949, '51, '53, '55	88
Jan	1949, '51, '53, '55, '57	85	Jan	1950, '52, '54, '56	72
Feb	1949, '51, '53, '55, '57	88	Feb	1950, '52, '54, '56	71

2. DATA USED FOR THE STUDY

Observations made at the Weather Bureau Airport Station in Phoenix during December, January, and February over a 9-year period, were used in making the comparisons. These data were divided into two groups as indicated in table 1.

Alternate years were used in each group to reduce the effects of any year-to-year persistence that might be present in the data. Only data for clear nights were used: a night was called "clear" if the mean sky coverage for the nine hourly observations (0030–0830 MST) was in the range 0–3/10. Hence, if any of the derived relationships are applied to operational forecasting, the forecaster is left to make the decision of whether or not the night will be clear.

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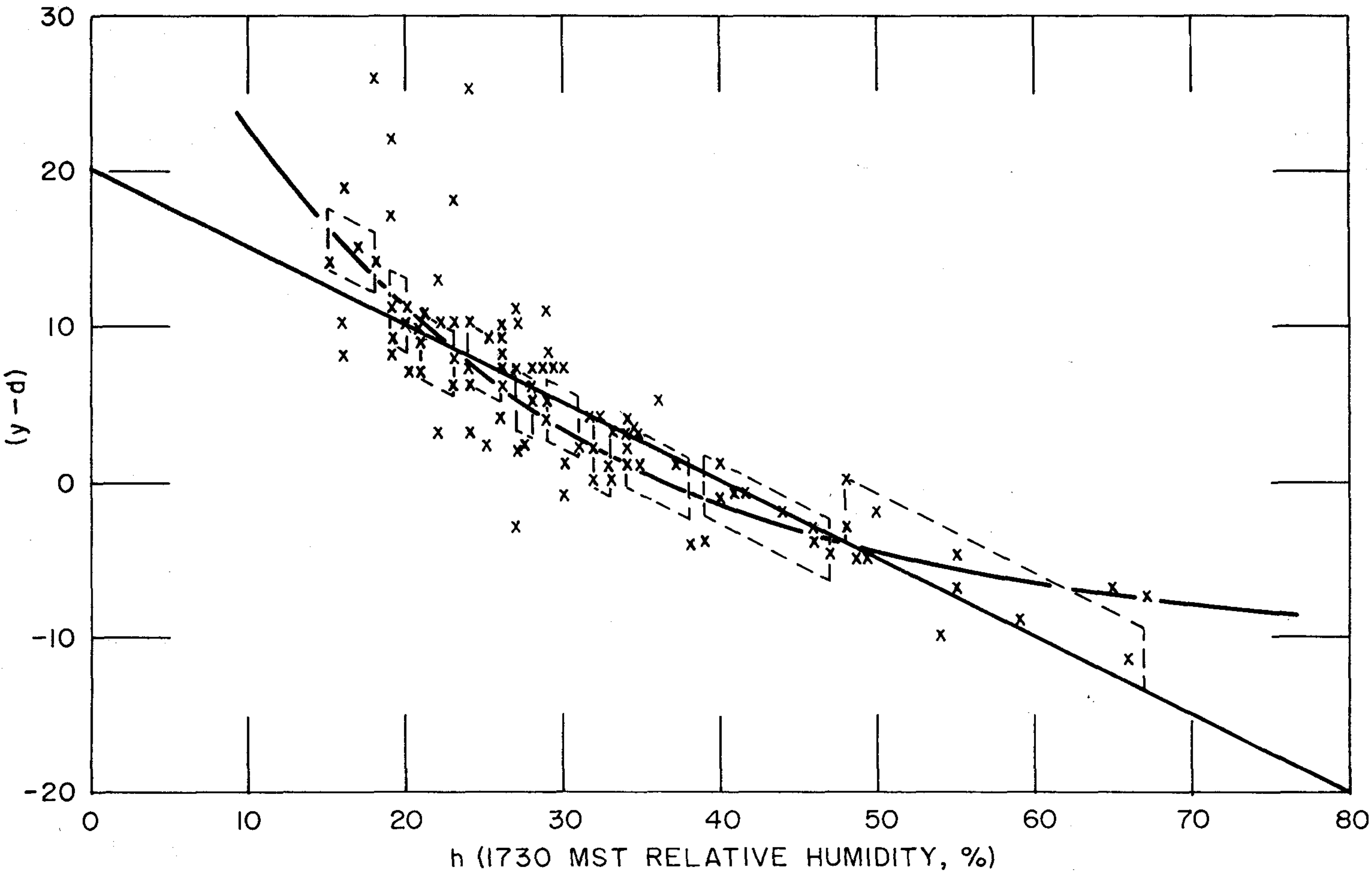

FIGURE 1.—Graph of $(y-d)$ against h for the December original data.

3. COMPARISON OF THE METHODS

YOUNG'S METHOD

Some years ago, Ellison [3] made a rather extensive comparison of a number of hygrometric formulas by applying them to the same set of data. On the basis of that study, he concluded that the Young formula gave the best results, especially if modified by the "method of arbitrary corrections." Therefore, the first method tested in the present study was Young's formula based on local data.

Individual Young formulas for clear nights were constructed using the original data for December, January, and February, respectively. Complete instructions for the development of the relationship are given in [3]; however, a brief description of the procedure used will be given here. First a graph was plotted using $(y-d)$ as the ordinate and h as the abscissa, where h is the 1730 MST relative humidity percent, d is the 1730 MST dew point (°F.), and y is the observed minimum temperature (°F.) the following morning. A straight line of best fit (minimizing the absolute error) was then fitted by inspection. Figure 1 shows the graph for the December original data. Similar graphs prepared for January and February are not reproduced here.

Next the equation of the straight line was written in the form recommended in [3], known as the "Donnel formula,"

$$y=d-\frac{h-n_1}{n_2} \tag{1}$$

where n_1 is the h-intercept and n_2 is the *negative* of the reciprocal of the slope of the line.

The final form recommended [3] for the equation is

$$y=d-\frac{h-n_1}{n_2}+V_d+V_h \tag{1A}$$

where V_d and V_h are correction factors derived from the original dew point and relative humidity data, respectively, as follows:

1. Values of d and h for each day were written on cards, along with the date.
2. Using the straight line formula (1), the formula minimum temperature was calculated, and the departure between this and the observed minimum temperature was written on each card.
3. The cards were then arranged in the order of increasing relative humidity. At intervals of approximately 10 cards, the average departure of the formula minimum

TABLE 2.—*Formula for the minimum temperature, y, for December, and set of arbitrary corrections*

$$y=d-\frac{h-40}{2}+V_d+V_h$$

h	V_h	d	V_d
15–18	+3.5	13–19	+1.5
19–20	+1.5	20–24	−0.5
21–23	−0.5	25–28	+0.5
24–26	+0.5	29–30	−0.5
27–28	−1.0	31	+1.5
29–31	−0.5	32–33	−0.5
32–33	−2.0	34	0
34–38	−1.0	35–36	+0.5
39–47	−0.5	37–38	−2.5
48–67+	+2.5	39–46+	−0.5

temperature estimate from the observed minimum temperature was calculated. This was then used as the arbitrary correction to be applied to the formula over the range of relative humidity indicated by the group of cards. This correction was applied to the original formula estimate and a new departure was calculated.

4. After the relative humidity corrections were determined and applied, the cards were arranged in the order of increasing dew point. The process just outlined was repeated, and the arbitrary correction to be applied to the formula over a given range of dew point was computed.

The formula for December and the set of arbitrary corrections are given in table 2. Similar formulas were derived for January and February. In applying the formulas to the data it is not necessary to solve the equations each time, as it is simpler to prepare a table of values of $(y-d)$ against h from the straight line, then add the corrective factors algebraically to the tabular value. This procedure is especially helpful if a curvilinear relationship is drawn to the data as on figure 1.

From these relationships, the minimum temperature was calculated on clear nights, using the original data and the independent data. The average absolute forecast error for each group is shown in table 4. One set of values was computed using the straight line formulas alone (without the correction factors V_d and V_h) and another set was computed using the complete relationship. The former are labeled "uncorrected" and the latter "corrected." Notice that the application of the method of arbitrary corrections to the original data produced a marked increase in skill but that no increase resulted when the method was applied to test data.

Next a curved line was fitted by inspection to the same data for each month (the line for the December data is shown in figure 1). The results obtained by testing each of these lines on both original and test data are given in table 4 under the heading, "Young's method, curved lines, corrected." Again, the method of arbitrary corrections was applied to forecasts made from these lines, but this time the corrections were rounded off to the nearest whole degree. The correction values for the December data only are shown in table 3. Again no significant increase

TABLE 3.—*Correction values for Young's method, curved lines, December data*

h	V_h	d	V_d
15–19	0	13–19	+1
20–22	−2	20–24	0
23–24	+1	25–28	0
25–26	+1	29–30	0
27–28	+1	31	+1
29–31	+1	32–33	0
32–33	0	34	0
34–38	+1	35–36	+1
39–47	0	37–38	−2
48–67+	0	39–46	−1

occurred when the corrections were applied to test data (see table 4 under heading, "Young's method, curved lines, corrected").

To understand this lack of improvement from the use of the method of arbitrary corrections, one must understand what the method does. As pointed out in [8], application of the method to all possible combinations of values of d and h, using finite intervals of these, yields a series of straight lines that are always parallel to the original straight line. In [8] adjacent end-points of the intervals were connected with straight lines, thereby giving a continuous, irregular line. Such a procedure has no meaning, however, because there is no way equation (1A) can generate solutions for line segments with slopes different from that of the straight line given by the part of the formula without the correction factors. Thus a series of disconnected straight lines should be drawn.

Instead of lines for all possible combinations of V_d and V_h, only the top and bottom of the series of lines were entered in figure 1, so that the areas within the rectangles contain all solutions possible with the December formula and tabulated corrections. The top of the farthest rectangle to the left (corresponding to $15 \geq h \geq 18$) is obtained from the December formula using $V_h=3.5$, $V_d=1.5$; while the bottom is obtained using the lowest tabulated value for V_d (-2.5). The rectangles for the other intervals of h and d used in the table were drawn on figure 1 in the same fashion.

Notice that the general pattern of the rectangular areas indicates a curvilinear relationship that follows somewhat the curved line fitted by inspection, but there is considerable up and down fluctuation which follows rather faithfully the sampling variation in the original data. This same sampling variation is then imposed on the test sample, which may explain the fact that no increase in skill was obtained over the linear relationship; applying this original sampling variation to the test sample nearly counterbalances the increase in skill contributed by the general curvilinear pattern followed by the areas. Obviously, correction for the sampling variation can cause a significant increase in skill only if the method is tested on the original data alone as was done in [3].

Smith [10] suggested that a curve of the second degree, drawn on the graph of $(y-d)$ against h, will probably yield better results, in the long run, than the Young formula. This seems borne out by the verification scores

TABLE 4.—*Comparison of average absolute error (° F.) obtained on samples of original and test data using the various methods*

	Young's method (straight lines)				Young's method (curved lines)				Jacob's method		Wet bulb vs. dry bulb method	
	Uncorrected		Corrected		Uncorrected		Corrected					
	Original data	Test data	Original data	Test data	Original data	Test data	Original data	Test data	Original data	Test data	Original data	Test data
December	2.7	3.3	2.4	3.2	2.7	2.9	2.5	2.8	3.2	3.7	2.3	2.7
January	3.6	3.5	2.7	4.0	3.1	2.9	2.5	3.0	4.4	3.0	2.2	2.7
February	3.3	3.5	2.5	3.0	2.5	2.8	2.1	2.7	3.8	3.9	2.3	2.2
Average	3.2	3.4	2.5	3.4	2.8	2.8	2.3	2.8	3.8	3.6	2.3	2.5

in table 1, because the curved line *without* correction factors gives a higher score on test data than the linear relationship *with* the correction factors. Note also that there is little difference between the scores on original and test data for the curved line, and on the test data for the curve with correction factors applied; however, the correction factors applied to the original data improved the skill considerably. This indicates that the correction factors are primarily correcting for the sampling variation in the original data, and that the curve is doing as good a job by itself of estimating the relationship as the curve with the factors applied.

JACOBS' METHOD

Another aid to forecasting the minimum temperature, used with some success in the past, is Jacob's [6] diagram, which is based on a formula for the temperature change at a key station in El Centro, Calif.:

$$\Delta t_1 = 12.32(0.56 - 0.08\sqrt{e})\sqrt{t} \tag{2}$$

where e is the vapor pressure (mb.), Δt_1 is the temperature drop (° C.), and t is the time (hours) after sunset (or length of night). This formula was derived, in turn, from a formula by Brunt [1]

$$y = T_o - \frac{2\sigma T^4}{\sqrt{\pi}}\left(\frac{1-a-b\sqrt{e}}{\rho_1 c_1\sqrt{k_1}}\right)\sqrt{t}. \tag{3}$$

In (3), ρ_1 is the soil density, c_1 is the specific heat of the soil, and k_1 is the thermal conductivity of the soil, σ is Stefan's constant, T is the radiative temperature (mean temperature during the night, for practical purposes), T_o is the temperature at sunset, and y is the forecast minimum temperature.[1] Jacobs [6] constructed a set of graphs based on formula (2) and provided factors to correct for variations in T and for different values of the "effective" soil factor (s), where $s = \sqrt{\pi}\rho_1 c_1\sqrt{k_1}$. a and b are constants in an empirical equation developed by Brunt [1] which expresses the relationship between the surface vapor pressure, the total long-wave radiation downward from the atmosphere, and the total black-body radiation at a given surface temperature on clear nights. The values of a and b used here are those used by Jacobs ($a = 0.44$, $b = 0.08$).

[1] Jacobs used T_1 for the minimum temperature, but y will be used here to be consistent with the notation used above.

Local average values of $\rho_1 = 2.70$ gm. cm.$^{-3}$ and $c_1 = 0.20$ cal. gm.$^{-1}$ °C.$^{-1}$ were obtained from the College of Agriculture at Arizona State University. The mean value of $k_1 = 4.7 \times 10^{-4}$ cal. sec.$^{-1}$ cm.$^{-1}$ °C.$^{-1}$ given by Jacobs [5] was used, along with a value of $t = 14.0$ hr. The shelter temperature at 1730 MST was substituted for T_o and the vapor pressure at the same time was used for e.

In computing formula (2), Jacobs used a value of $T = 280°$ AA. In the present study, a value of $T = 283°$ AA was used since the mean nighttime temperature at Phoenix during the winter averages near 50° F. Actually, the numerical value of T used in developing the basic formula is arbitrary, since a correction is ultimately included for the observed (or forecast) mean temperature during the night; it becomes important only if the correction is not applied.

The values of the variables and constants given above, gave the following formula for Phoenix:

$$T_o - y = 16.1(0.56 - 0.08\sqrt{e})\sqrt{t}. \tag{4}$$

This equation expresses $(T_o - y)$ in degrees Celsius. Curve A on figure 2 gives $(T_o - y)$ in degrees Fahrenheit, using a constant value of $t = 14.0$ hours. Figure 3 gives the value of the correction factor (f) by which the value of $(T_o - y)$ from figure 2 must be multiplied to correct for the actual mean temperature during the night. In the verification, the observed value of $T = \frac{1}{2}(T_o + y)$ was used as the mean temperature during the night. Formula (4) was verified only on the December original data, since it became obvious during the verification that the formula was consistently forecasting minimum temperatures that were much too low. An average error of 11.5° F. was obtained.

Brunt [1] states that the chief difficulty in the way of using formula (3) for forecasting the night minimum temperature lies in the uncertainty of the value of the coefficient $\rho_1 c_1\sqrt{k_1}$, and that the addition of 20 percent of water to dry soil increases this factor five-fold. Jacobs [6] suggests that these factors can probably best be derived empirically, so equation (3) was written in the form

$$\frac{1}{\rho_1 c_1\sqrt{k_1}} = -\frac{\dfrac{\sqrt{\pi}}{2\sigma T^4\sqrt{t}}(y - T_o)}{(1-a) - b\sqrt{e}}. \tag{5}$$

From the values stated above of σ, T, t, a and b, and observed values of y, T_o, and e (1730 MST) for all clear nights

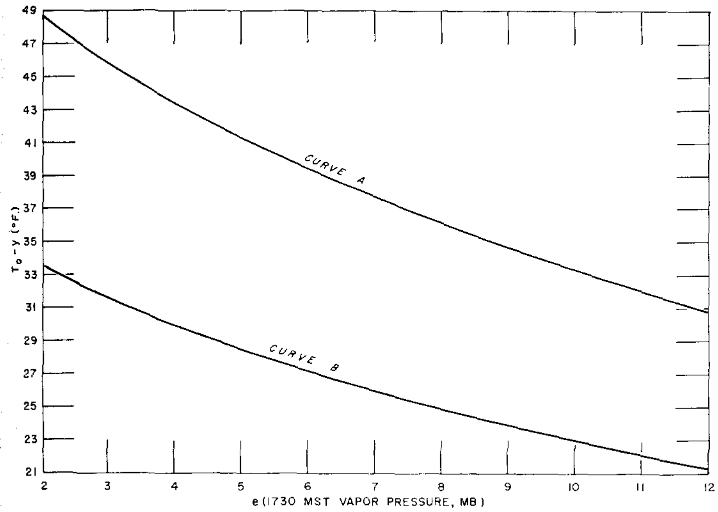

FIGURE 2.—$(T_o - y)$ plotted against vapor pressure (e) at 1730 MST. Curve A=curve for equation (4). Curve B=curve for equation (6).

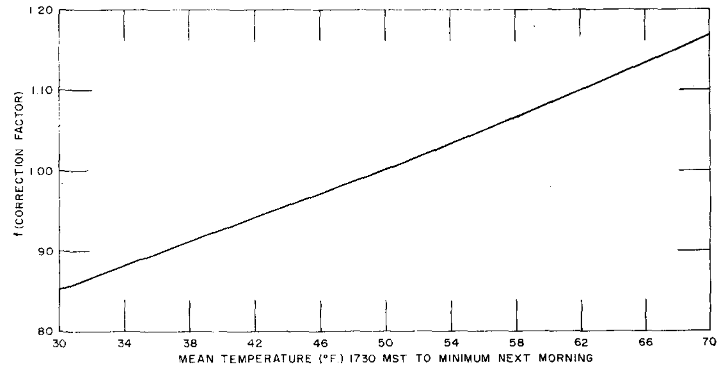

FIGURE 3.—Correction factor (f) by which $(T_o - y)$ from equations (4) and (6) should be multiplied to correct for the mean temperature during the night.

in December, January, and February of the original data, a value of $1/\rho_1 c_1 \sqrt{k_1}$ was computed for each of the 271 nights. A frequency polygon was constructed from these values and the median value computed. The distribution of values turned out to be very regular (little or no skewness) so that the mean and the median coincided at 33.5. From the frequency polygon, a percentile estimate of the standard deviation using 10 percentiles [2] gave a value of 7.5. This estimate (statistical efficiency 0.92) of the standard deviation is included only to give some idea of the spread of the data about the mean value.

The median value of $1/\rho_1 c_1 \sqrt{k_1}$ was then substituted into equation (3) and by rearrangement a new equation similar to (4) was obtained:

$$T_o - y = 11.1\ (0.56 - 0.08\sqrt{e})\ \sqrt{t}. \qquad (6)$$

Like equation (4), this equation gives $(T_o - y)$ in degrees Celsius, assuming a mean temperature during the night of 283° AA. Curve B on figure 2 gives expected values of $(T_o - y)$ in degrees Fahrenheit, using a constant value of $t=14.0$ hr. Figure 3 may be used in the same manner as with equation (4) to correct for the observed mean temperature during the night.

As in the case of equation (4), the verification of this equation was carried out using observed values of $T=\frac{1}{2}(T_o+y)$. As can be seen in table 4, the results are considerably improved over those of equation (4). The use of Jacobs' equation on an operational basis requires, in addition to a forecast of sky condition, an estimate of the mean temperature during the night; however, using the observed 1730 MST temperature and a rough estimate of the minimum temperature to compute $T=\frac{1}{2}(T_o+y)$ would probably have given results on the same data nearly comparable to those obtained here.

A MORE DIRECT APPROACH

A climatological aid for forecasting the minimum temperature should be a simple relationship based on evening data, which the forecaster can use for guidance in making his forecast. Most of the methods developed in past years have used evening temperature and moisture parameters to forecast the minimum temperature the following morning, and physical justification for this procedure can be found in Brunt's equation. In equation (3) all variables are assumed to be constants with the exception of y, T_o, T, t, and e. We have seen, however, that T may be closely approximated by $(y+T_o)/2$, and t may be taken as a constant (14.0 hr. was used above). Hence, the only variables left are y, T_o, and e. Young's method essentially substitutes two other functions of T_o and e; namely, h and d. Another possibility for a 3-variable combination that seems quite feasible, in terms of Brunt's equation, is y, T_o, and the wet bulb temperature T_w. Both T_o and T_w are measured directly in the instrument shelter and so are somewhat easier to use on an operational basis than d and h.

Given three variables, T_o, T_w, and y, between which a joint relationship is suspected, there are a number of ways to approximate this relationship [4], [7]. One of the most straightforward, however, is to plot T_o along one coordinate axis, T_w along the other, and the numerical value of y at the intersection point in the field of the graph corresponding to particular values of T_o, T_w. Isopleths of constant values of y may then be drawn by one of several methods [4], [7]. A diagram of this type is shown in figure 4, based on the 271 cases of original data for December, January, and February combined. The isopleths were drawn by the method of substratification and successive graphical approximation described in [4].

The advantage of this type of combination of variables is that no initial assumption is made about the form of the relationship, and the data are given freedom to indicate their relationship within the limitations of the sample size. This is in contrast to the Young formula, for example, which plots two variables along one axis as $(y-d)$, and thereby implicitly assumes that these two variables have equal weight in the relationship with h, and that the regression of y on d (or vice versa) is linear for constant h.

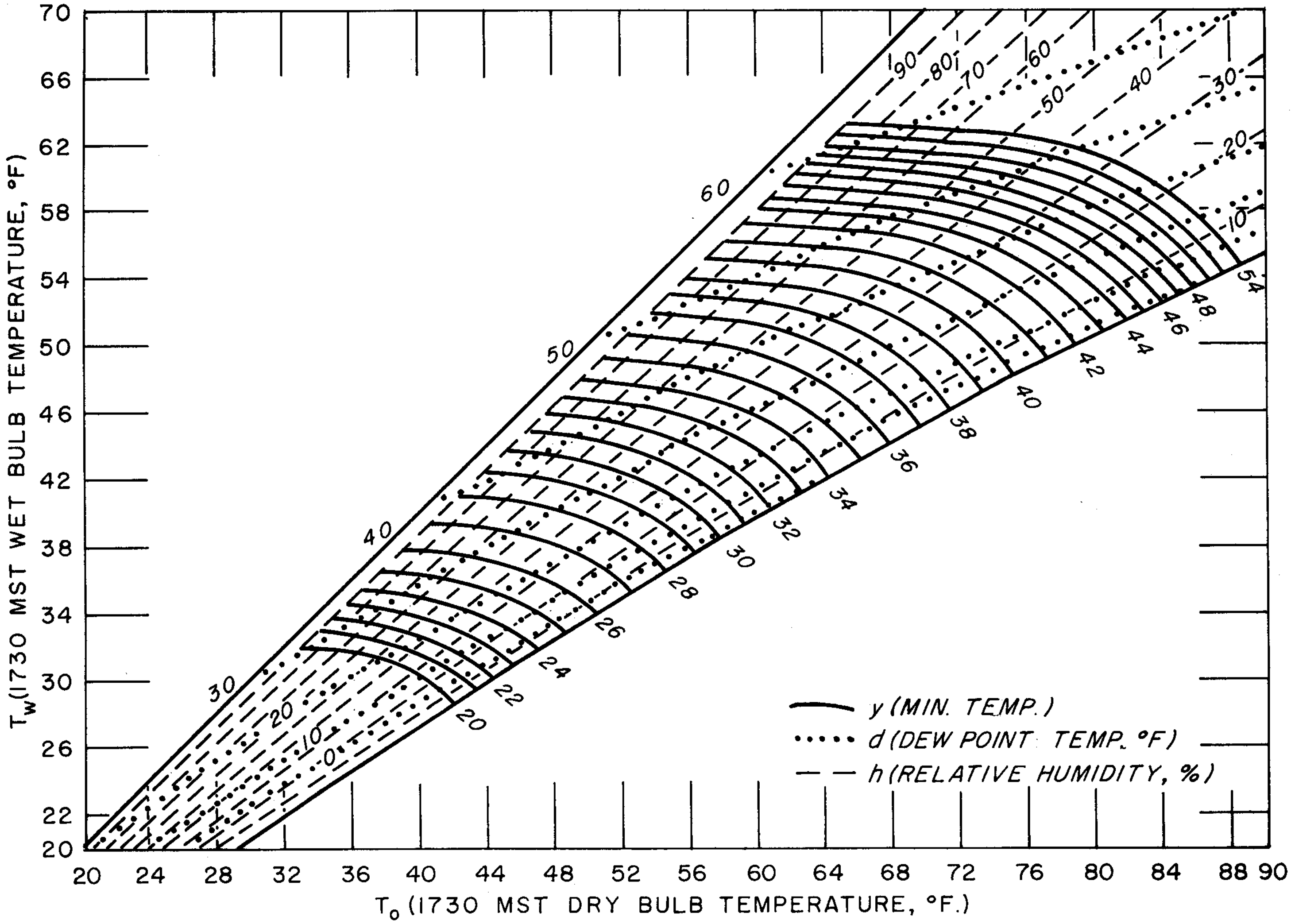

FIGURE 4.—Graph showing the estimated joint relationship between T_o, T_w, and y on clear nights. Lines of constant d and h were added using hygrometric table after analysis was performed.

These assumptions become readily apparent if the relationship between y, d, and h is examined on a graph using $(y-d)$ as the ordinate and h as the absicssa. Such a relationship can be expressed in the following general functional form,

$$y-d=f(h) \tag{7}$$

or,

$$y=f(h)+d \tag{8}$$

where $f(h)$ is assumed to be a continuous function for all values.

Now consider the effect of holding h constant,

$$y=\text{constant}+d. \tag{9}$$

The linearity between y and d for constant h and the assumption of equal weight of y and d with respect to h are shown by (9), and can be graphically illustrated on a diagram of the kind shown in figure 4, by using y as the ordinate, d as abscissa, and h in the field of the diagram. Such a diagram will always have the following set of characteristics no matter how complicated the curve drawn on figure 1 to indicate the relationship between y, d, and h, lines of constant h will always be straight lines, and the slope of these lines will always be 1. As an example, the relationship between y, d, and h shown by the curved line in figure 1, is converted into this kind of diagram in figure 5 (solid lines).

Since the Young method has the implicit assumptions mentioned above, it might be of interest to see how well these assumptions are satisfied by the analysis in figure 4, since that analysis was carried out unencumbered by those assumptions. To do this, however, we must convert the relationship shown in figure 4 into one between y, d, and h. This can be done simply by drawing lines of constant d and h in figure 4 using a hygrometric table. These lines can then be used to graph the relationship between y, d, and h in figure 5 (dashed lines), for a comparison with the lines (solid) drawn using the curvilinear relationship in figure 1. Obviously, such a comparison can be used only to indicate differences between the results of the two methods on this sample of data, since neither one is known to be "correct." The dashed lines, which were free to assume

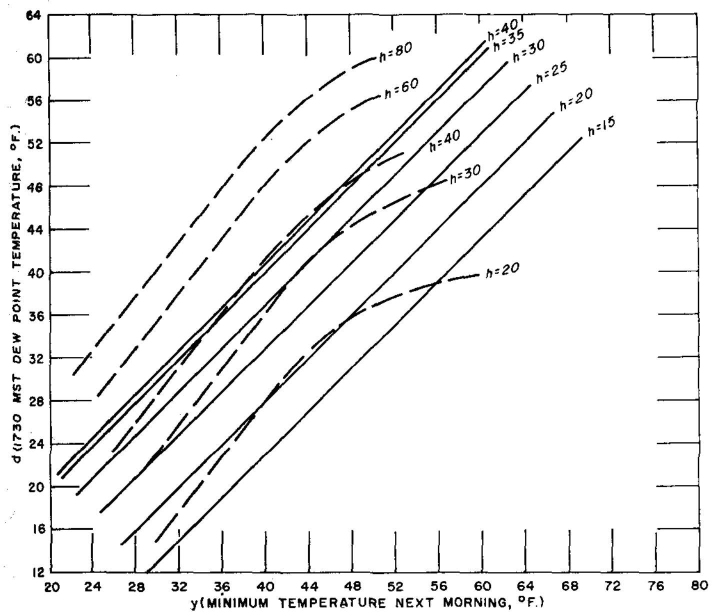

FIGURE 5.—Graph showing relationship between d, y, and h for clear cases. Solid lines show the relationship represented by the curve in figure 1. Dashed lines show the relationship from the analysis on figure 4.

any configuration in figure 5, seem quite straight for values of y less than about 40°. Thus the assumption of linearity of the regression of y on d for constant h seems fulfilled by the figure 4 analysis for lower values of y, but seems to break down at higher values of y (and higher values of d and h).

Young [12] found that forecasts made from a straight line relationship on a graph like figure 1 were considerably in error at high values of d and h (and also for very low values) and developed the method of arbitrary corrections to correct for this. It should be emphasized here, however, that the statement above about the indicated lack of linearity in figure 5, should not be confused with Young's finding. The lack of linearity indicated in figure 5 is between the regression of y on d for constant h, and if real, cannot be corrected by any method as long as the basic graph is like that used in figure 1. Obviously, the method of arbitrary corrections also operates within the framework of the two assumptions described by equation (9).

For the dashed lines in figure 5, the slope is approximately 1.3 for values of y less than about 40. For higher values of y, the slope becomes variable.

Thus the assumptions implicit in the Young method are only approximately fulfilled by the original analysis in figure 4. Although we cannot draw definite conclusions from the comparison in figure 5, it is possible that some of the difference in skill on the test data between the analysis in figure 1 and that in figure 4 (see table 4) is due to a lack of conformity of the data to the assumptions made in figure 1.

To provide some standard of comparison for the performance of figure 4, temperature estimates made using 1730 MST data were compared with official forecasts made by the staff at WBAS Phoenix. The official forecasts of the minimum temperature the following morning are made on an operational basis at 1930 MST each evening. The average absolute error by seasons is shown in table 5 for all clear cases. The seasons used were from the test sample for figure 4 (1954 season=Dec. 1953, Jan. and Feb. 1954; 1956 season=Dec. 1955, Jan. and Feb. 1956; etc.). It should be pointed out in this comparison, that the climatological aid "knew" that the night would be clear, while the staff forecasters did not.

TABLE 5.—*Comparison of average absolute error for the climatological aid (T_o vs. T_w) and official forecasts for clear nights*

Season	No. cases	T_o vs. T_w	Official forecasts
1954	63	2.7	3.8
1956	57	2.2	3.2
1958	53	2.2	2.2

It is of interest to note that the climatological aid averaged approximately 1° F. better than the official forecasts in the first two seasons, but in the last season the skill was about the same for both. During the 1958 season the Meteorologist in Charge placed the climatological aid in the station forecast manual and the forecasters used it on a discretionary basis. There is no way of determining what part (if any) of the improvement in skill shown in the 1958 season was due to its use.

4. CONCLUSIONS

YOUNG'S METHOD

In this method a graphical combination of y, d, and h is performed, but the combination is done within the framework of the following assumptions: (1) the regression of y on d, for constant h, is linear, and (2) y and d have equal weight in the relationship with h. If these assumptions are not fulfilled by the "population," a loss in skill can be expected over that obtained by a method of analysis allowing greater degrees of freedom.

Another logical combination of variables, in terms of Brunt's equation, is T_o, T_w, and y. An advantage of this combination is that the independent variables (T_o and T_w) are measured directly in the instrument shelter. These variables were combined on a graph using T_w as ordinate and T_o as abscissa. If estimates of y are desired in terms of d and h, as in Young's method, it is only necessary to add lines of constant d and h to this diagram with a hygrometric table.

With Brunt's equation as a working physical model, Young's method which combines h, d, and y is seen to be essentially the same as the method employing T_o, T_w, and y. It is possible that the improvement in skill shown on test data by the latter method is largely due to the use of an improved method of data analysis. Whether or not this is true, we have seen that the Young method of combining d, h, and y introduces assumptions for which no *a priori* physical reasons have been given. There seems little reason for imposing these assumptions today, when we have a number of good methods for estimating the joint relationship between several variables which do not require such arbitrary restrictions.

A relationship between y, d, and h was derived using a method of analysis that did not arbitrarily place these two restrictions on the data. The following conclusions were drawn for the particular analysis performed on this particular set of data: (1) assumption No. 1 was well satisfied for observed minima below about 40° F. but broke down at higher minima, and (2) assumption No. 2 was not quite as well satisfied, even at temperatures in the lower ranges.

If the Young method of combining y, d, and h is used, it is recommended that instead of preserving the original first-degree estimate of the plotted relationship between $(y-d)$ and h by applying the corrections in the form of formula (1A), a curvilinear relationship of the second degree be drawn. Forecasts made from this line will probably be subject to less loss in skill caused by imposing original-data sampling variation on future samples.

JACOBS' METHOD

This method does not seem to be more capable of universal application (application without determining local constants by data processing) than other methods. This may be due primarily to the difficulty of obtaining reliable estimates of the soil factors, which probably not only undergo considerable seasonal variation, but also may differ, on the average, between clear and cloudy nights.

A MORE DIRECT APPROACH

Inspection of Brunt's equation (3) provides a physical justification for using the evening dry bulb temperature (T_o) and the evening vapor pressure (e) to forecast the minimum temperature the next morning (y). Since dew point (d) and relative humidity (h) are functions of T_o and e, the combination of d, h, and y also seems logical, but it is recommended that a method of estimating the joint relationship between these variables be used that does not place unnecessary restrictions on the data.

5. SUGGESTIONS FOR FURTHER RESEARCH

This study has discussed methods which, in effect, use T_o and e to estimate y. Obviously, there are many other variables of great importance, but the introduction of these into the relationship was beyond the scope of this project. Some of these important additional variables are (1) wind, (2) turbulence, (3) advection, (4) soil moisture, (5) soil temperature, (6) air temperature aloft, and (7) moisture distribution aloft. One approach to future research in this field might be to incorporate these variables into the relationship, perhaps using the "residual method" [9], which has the advantage of showing whether an additional variable improves a forecast based on other variables.

Another approach may be to work directly with Brunt's expression for the fall in temperature after sunset, which is

$$\frac{2}{\sqrt{\pi}}\,\frac{R_N}{\rho_1 c_1\sqrt{k_1}}\sqrt{t}$$

where R_N is net radiation, and t is the time after sunset. Since changes in soil moisture are mainly responsible for changes in the factor $\rho_1 c_1\sqrt{k_1}$ in a given locality, it may be possible to substitute a soil moisture reading, empirically, for this factor. R_N can be measured using fairly inexpensive equipment [11]. One procedure would be to take a reading of R_N near or shortly after sunset and, following Brunt's suggestion, consider it to be constant throughout a given clear night. Forecasts made in this fashion would have to be modified subjectively for advection, wind, and turbulence, just as those discussed previously.

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APPENDIX

EXTENSION OF THE METHOD IN FIGURE 4 TO CLOUDY NIGHTS

To extend the usefulness of the climatological aid, graphs similar to figure 4 were prepared for two other classes of nights, as follows:

Class 1. This class contains principally nights with high broken to overcast conditions and with no persistent ceiling below 10,000 feet. Criteria: on the morning following the evening on which the forecast was made, the observed mean sky coverage on the nine hourly observations 0030–0830 MST, inclusive, was in the range 4/10–10/10 *and* a ceiling of 10,000 feet or less was reported on no more than 4 of these observations.

Class 2. This class contains principally nights with broken to overcast skies and with a persistent ceiling below 10,000 feet. Criteria: on the morning following the evening on which the forecast was made, the observed mean sky coverage on the nine hourly observations 0030–0830 MST, inclusive, was in the range 4/10–10/10 *and* a ceiling of 10,000 feet or less was reported on 5 or more of these observations.

It will be noted that on figure 4 the minimum temperature lines are not extended into the region of the graph above a relative humidity of 90 percent. Extension of the lines into that region would have little meaning because a relative humidity that high in the early evening usually leads to ground fog by morning, automatically placing the case in the "cloudy, class 2" category. For this same reason there were practically no observed cases in this region of the diagram for the "cloudy, class 1" category, (fig. A–1) so the minimum temperature lines above a relative humidity of 90 percent were omitted on this diagram also.

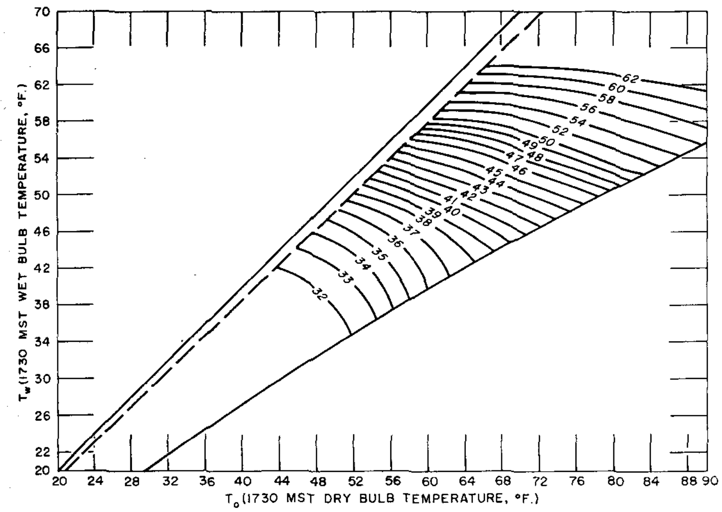

FIGURE A-1.—Graph showing the estimated joint relationship between T_o, T_w, and y on cloudy nights (class 1). Lines of constant y (estimated) are drawn in the field of the diagram.

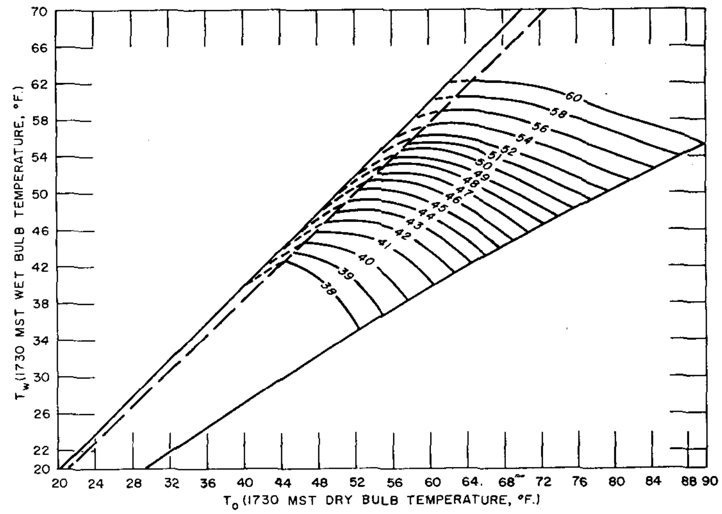

FIGURE A-2.—Graph showing estimated joint relationship between T_o, T_w, and y for cloudy nights (class 2). Lines of constant y (estimated) are drawn in the field of the diagram.

On the "cloudy, class 2" diagram (fig. A-2), the minimum temperature lines were drawn from the intersection with the 90 percent relative humidity line to a point of intersection with the 100 percent relative humidity line, corresponding to a dry bulb temperature about 2° higher than the expected minimum temperature line. This was an arbitrary decision, based on the assumption that if the relative humidity were 100 percent at 1730 MST, the dry bulb temperature would only fall about 2° during the night. This decision was based on a few observed cases on foggy nights, but the frequency of such nights in Phoenix is very low so the evidence for determining the configuration of the lines in this region of the chart was flimsy. Physical reasoning would indicate, however, that as the relative humidity approaches 100 percent the release of the latent heat of condensation, as fog formation begins and progresses, should reduce the rate of temperature drop materially, and the minimum temperature lines at high relative humidities should be drawn with that fact in mind. Enough data might be available at coastal stations to do a much better job of analysis in that region of the chart.

Because of the fewer number of cases in the two cloudy categories, the original data sample was extended to include the following years: December 1948–1955; January and February 1949–1956. This gave 175 cases in class 1, and 85 cases in class 2. The verification on original data is shown in table A-1.

The verification on test data was carried out for December 1956, 1957; January and February 1957, 1958 (a total of 64 cases for class 1 and 21 cases for class 2). Because of the scarcity of cases, the verification in table A-1 is given only for the whole test sample for both the climatological aid and the official forecasts.

The importance of being able to forecast cloudiness during the night is especially apparent in the great differences in skill between the climatological aid and the official forecasts for class 2, since the former essentially "knew" the ensuing sky conditions at the time of "forecast."

TABLE A-1.—*Average absolute error for cloudy cases using original data and comparison with official forecasts using test data*

Month	Years used	Original data				Test data					
		Class 1		Class 2		Class 1			Class 2		
		No. cases	Avg. abs. error	No. cases	Avg. abs. error	No. cases	T_o vs. Tw	Off. forecasts	No. cases	T_o vs. Tw	Off. forecasts
Dec	48–55	56	3.0	28	2.3	64	3.0	3.7	21	3.6	6.4
Jan	49–56	65	3.0	36	2.5						
Feb	49–56	54	3.1	21	2.6						
Average		175	3.0	85	2.5						

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